In exercises 1 – 9, evaluate the limit.

1. \( \lim_{x \to -2} \frac{x + 1}{x^2 + x} = 0 \)
2. \( \lim_{x \to 0} \frac{\sin 3x}{5x} = \frac{3}{5} \)
3. \( \lim_{x \to 0} \frac{x + \sin x}{x} = 2 \)
4. \( \lim_{x \to 1} \frac{4x^2 + 3x - 2}{3x^2 - 7} = \frac{4}{3} \)
5. \( \lim_{x \to \frac{\pi}{2}} \csc x = \frac{2}{\sqrt{3}} \)
6. \( \lim_{x \to \frac{1}{2}} \frac{4x^2 - 2x}{8x - 4} = \frac{1}{4} \)

In exercises 10 and 11, find all horizontal asymptotes, vertical asymptotes, and removable discontinuities. Justify using limit statements.

10. \( f(x) = \frac{3x^2(2 - x)}{3x^2} \)
   - HA: \( y = -1 \)
   - V.A: \( x = 0 \)
   \( \text{no removable discontinuities} \)

12. Let \( f(x) = \begin{cases} x - 2, & \text{if } x < 0 \\ x^2 - x, & \text{if } 0 \leq x \leq 3 \\ 3x - 2, & \text{if } x > 3 \end{cases} \)
   (a) Graph the function to the right.
   (b) \( \lim_{x \to -2} f(x) = \frac{10}{2} = 5 \)
   (c) \( \lim_{x \to 0} f(x) = \frac{1}{2} \)
   (d) \( \lim_{x \to +3} f(x) = 16 \)
   (e) Is the function continuous for its entire domain? Justify using the definition of continuity.
   \( f(x) = \begin{cases} -1, & \text{if } x \leq -1 \\ 1, & \text{if } -1 < x < 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } 0 < x < 1 \\ 0, & \text{if } x \geq 1 \end{cases} \)
   (a) Graph the function to the right.
   (b) \( \lim_{x \to -1} f(x) = -1 \)
   (c) \( \lim_{x \to 0} f(x) = \text{DNE} \)
   (d) \( \lim_{x \to 1} f(x) = -1 \)
   (e) Is the function continuous? Justify using the definition of continuity.
   \( \text{not continuous; removable discontinuity at } x = 0 \)
   \( \text{jump discontinuity at } x = 1 \)

In exercises 14 and 15, find the value of \( k \) that makes the function continuous everywhere.

14. \( f(x) = \begin{cases} kx^2, & \text{if } x < 2 \\ 2x + k, & \text{if } x \geq 2 \end{cases} \)
   \( k = \frac{1}{2} \)

15. \( f(x) = \begin{cases} 2x^2 - 5x - 12, & \text{if } x < 2 \\ kx^3, & \text{if } x \geq 2 \end{cases} \)
   \( k = -1 \)

In exercises 16 – 22, use the graph of \( g \) to answer the following questions.

16. \( \lim_{x \to -2} g(x) = -4 \)

17. \( \lim_{x \to 1} g(x) = 0 \)

18. \( g(1) = 1 \)

19. \( \lim_{x \to 2} g(x) = -2 \)

20. \( \lim_{x \to -2} g(x) = -2 \)

21. \( \lim_{x \to 2} g(x) = 1 \)

22. At what \( x \)-value(s) does \( f(x) \) appear to fail to be differentiable? \( x = -1 \)

In exercises 23 and 24, use the definition of the derivative, \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \), to find the derivative.

23. \( f(x) = x^2 - 6x \)
   \( f'(x) = 2x - 6 \)

24. \( f(x) = \frac{1}{x + 1} \)
   \( f'(x) = \frac{1}{(x + 1)^2} \)

In exercises 25 – 27, consider the function \( f(x) = \sin \left( \frac{x}{3} \right) \).

25. Find the average rate of change over the interval \([\pi, 2\pi]\). 0

26. Find the instantaneous rate of change at \( x = 2\pi \).

27. Write the equation for the tangent line to the curve for \( x = 2\pi \).

28. Let
   \( f(x) = \begin{cases} x^2 - 2x + 1, & \text{if } x \neq -2 \\ 0, & \text{if } x = -2 \end{cases} \)
   Determine if the function is continuous everywhere. Justify using the definition of continuity.

   \( \text{no, there is an infinite discontinuity at } x = -2 \)
In exercises 29 – 31, suppose that \( u \) and \( v \) are differentiable functions at \( x = 2 \), and that \( u(2) = 3 \), \( v(2) = -4 \), \( u'(2) = 1 \), and \( v'(2) = 2 \). Use this information to find the following derivatives.

29. \( \frac{d}{dx} [u(v)] \bigg|_{x=2} = 2 \)
30. \( \frac{d}{dx} [u(v)] \bigg|_{x=2} = 10 \)
31. \( \frac{d}{dx} [3u - 2v + 2uv] \bigg|_{x=2} = -12 \)

In exercises 32 – 37, use the following table to answer the questions.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( F(x) )</th>
<th>( F'(x) )</th>
<th>( G(x) )</th>
<th>( G'(x) )</th>
<th>( H(x) )</th>
<th>( H'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>4</td>
<td>-3</td>
<td>2</td>
<td>7</td>
<td>-2</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>6</td>
<td>10</td>
<td>-6</td>
<td>-4</td>
<td>11</td>
</tr>
</tbody>
</table>

32. If \( H(x) = (F(x))^2 \), then what is \( H'(3) \)?
33. If \( H(x) = \frac{F(x)}{G(x)} \), then what is \( H'(3) \)?
34. If \( H(x) = F(x) \cdot G(x) \), then what is \( H'(3) \)?
35. If \( H(x) = G(F(x)) \), then what is \( H'(3) \)?
36. If \( H(x) = G(F(x)) \), then what is \( H''(3) \)?
37. If \( H(x) = \ln(F(x)) \), then what is \( H''(3) \)?

In exercises 38 – 43, use the function \( f(x) = x^3 \) to answer the questions.

38. Find \( f'(x) \).
39. Evaluate \( f(2) \). Explain the meaning of this.
40. Find the average rate of change over the interval \([-2, 2] \) at \( x = 2 \).
41. The instantaneous rate of change of \( x^2 \) is \( 12 \); the slope of the tangent.
42. Find \( f''(x) \).
43. In exercises 44 and 45, use the table to answer the questions.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

44. Using the values in the table, estimate \( f''(3) \).
45. Use the data in the table to estimate the value of \( f''(5) \).
46. Explain why there must be a value \( c \), \( 5 < x < 10 \), such that \( f(x) = 0 \).

In exercises 47 – 64, find the derivative.

47. \( f(x) = -\frac{x^4 + 4}{x^5 + 3x + 4} \)
48. \( f(x) = 2x - \frac{2x^3 + \sqrt{1+x^7}}{4x} \)
49. \( f(x) = \frac{3 - 2x - x^2}{x^5 - 1} \)
50. \( f(x) = x^7 \sin(4x) \)
51. \( f(x) = y^3 \sin(4y) \)
52. \( h(x) = \cos(7x) \)
53. \( y = 2 \cos^3(3x + 1) \)
54. \( y = 2 \cos^3(3y + 1) \)
55. \( y = y'(x^2 - 3)^3 \)
56. \( y = \csc^2(2x^3) \)
57. \( f(x) = x^3 \tan^2 x \)
58. \( f(x) = \ln(1 + e^{\cos^2 x}) \)
59. \( y = \ln(x^4 + 3) \)
60. \( y = e^{x^2} \)
61. \( g(t) = \tan^{-1}(3t - 4) \)
62. \( h(x) = \frac{x}{1 - x^2} + \sin^{-1}(x) \)
63. \( f'(x) = \frac{e^{-x}}{1 + e^x} \)
64. \( y = \arcsin(5x) \)
65. \( f(x) = 4x^2 \cos(4x) + 2x \sin(4x) \)
66. \( h'(x) = \frac{7x \sin(7x) - 4 \cos(7x)}{4x^2} \)
67. \( f(x) = \frac{2x^2}{(1-x)^3} \)
68. \( y' = -24 \cos^3(3x+1) \sin(3x+1) \)
69. \( y' = (x^2 - 3)^2 (8x^5 - 2x) \)
70. \( y' = -12 \csc^3(2x^2) \cot(2x^2) \)
71. \( f(x) = 2x^3 \tan(x) \sec(x) + 3x^2 + 9x^2 \)
72. \( f(x) = 3e^{-3x} \)
73. \( y' = \frac{3x+3}{x^2 + 3} \)
74. \( y' = (2x - 3)e^{x^2 - 3x} \)
75. \( y' = (2x - 3)e^{x^2 - 3x} \)