Name: AP Calculus/Period Date

1st 9 Weeks Test Review

In exercises 1 – 9, evaluate the limit.

1. $\lim\limits_{x\to\infty}\dfrac{x+1}{x^2+3}$ 0

2. $\lim\limits_{x\to0}\dfrac{\sin 3x}{5x}$ $\frac{3}{5}$

3. $\lim\limits_{x\to0}\dfrac{x+\sin x}{x}$ 2

4. $\lim\limits_{x\to\infty}\dfrac{4x^2+3x-2}{3x^2-7}$ $\frac{4}{3}$

5. $\lim\limits_{x\to\frac{2\pi}{3}}\csc x$ $\frac{2}{\sqrt{3}}$

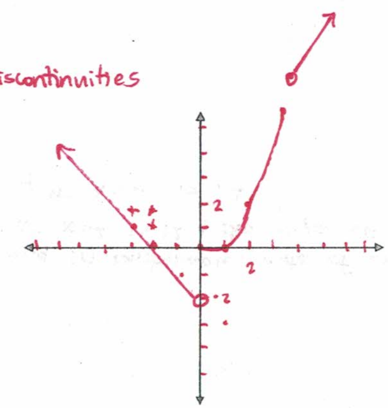
6. $\lim\limits_{x\to\frac{1}{2}}\dfrac{4x^2-2x}{8x-4}$ $\frac{1}{4}$

In exercises 10 and 11, find all horizontal asymptotes, vertical asymptotes, and removable discontinuities. Justify using limit statements.

10. $f(x)=\dfrac{3x^4(2-x)}{3x^5}$

H.A. $y=-1$
V.A. $x=0$
no removable discontinuities

12. Let $f(x)=\begin{cases}-x-2, & \text{if } x<0\\ x^2-x, & \text{if } 0\le x\le 3\\ 3x-2, & \text{if } x>3\end{cases}$



(a) Graph the function to the right.

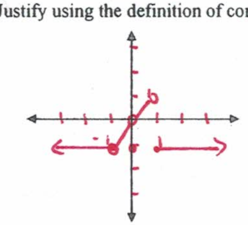
(b) $\lim\limits_{x\to-4}f(x)=$ 10

(c) $\lim\limits_{x\to1}f(x)=$ 1

(d) $\lim\limits_{x\to6}f(x)=$ 16

(e) Is the function continuous for its entire domain? Justify using the definition of continuity.

13. Let $f(x)=\begin{cases}-1, & \text{if } x\le-1\\ x, & \text{if } -1<x<0\\ -1, & \text{if } x=0\\ x, & \text{if } 0<x<1\\ -1, & \text{if } x\ge1\end{cases}$



(a) Graph the function to the right.

(b) $\lim\limits_{x\to2}f(x)=$ -1 (c) $\lim\limits_{x\to1}f(x)=$ DNE (d) $\lim\limits_{x\to-1^+}f(x)=$ -1

(e) Is the function continuous? Justify using the definition of continuity.

not continuous ; removable discontinuity at x=0
jump discontinuity at x=1

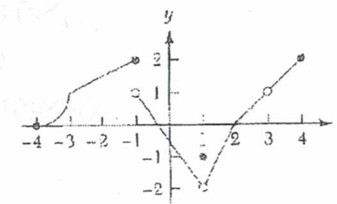
In exercises 14 and 15, find the value of k that makes the function continuous everywhere.

14. $f(x)=\begin{cases}kx^2, & \text{if } x<2\\ 2x+k, & \text{if } x\ge2\end{cases}$ $k=\frac{4}{3}$

15. $f(x)=\begin{cases}2x^2-5x-12, & \text{if } x<2\\ kx-13, & \text{if } x\ge2\end{cases}$ $k=-1$

In exercises 16 – 22, use the graph of g to answer the following questions.

Graph of g



16. $\lim\limits_{x\to-4}g(x)$ -4

17. $\lim\limits_{x\to2^-}g(x)$ 0

18. $g(1)$ -1

19. $\lim\limits_{x\to1}g(x)$ -2

20. $\lim\limits_{x\to-1^-}g(x)$ -2

21. $\lim\limits_{x\to3}g(x)$ 1

22. At what x-value(s) does $g(x)$ appear to fail to be differentiable? $x=-1$

In exercises 23 and 24, use the definition of the derivative, $f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$, to find the derivative.

23. $f(x)=x^2-6x$ $f'(x)=2x-6$

24. $f(x)=\dfrac{1}{x+1}$ $f'(x)=\dfrac{-1}{(x+1)^2}$

In exercises 25 – 27, consider the function $f(x)=\sin\left(\dfrac{x}{3}\right)$

25. Find the average rate of change over the interval $[\pi,2\pi]$. 0

26. Find the instantaneous rate of change at $x=2\pi$. $-\frac{1}{6}$

27. Write the equation for the tangent line to the curve for $x=2\pi$. $y=-\frac{1}{6}(x-2\pi)+\frac{\sqrt{3}}{2}$

28. Let

$$f(x)=\begin{cases}\dfrac{x^2-2x+1}{x+2}, & \text{if } x\ne-2\\ 3, & \text{if } x=-2\end{cases}$$

Determine if the function is continuous everywhere. Justify using the definition of continuity.

no, there is an infinite discontinuity at x=-2

In exercises 29 – 31, suppose that u and v are differentiable functions at $x = 2$, and that $u(2) = 3$, $u'(2) = -4$, $v(2) = 1$, and $v'(2) = 2$. Use this information to find the following derivatives.

29. $\left.\frac{d}{dx}[uv]\right|_{x=2}$ 2

30. $\left.\frac{d}{dx}\left[\frac{u}{v}\right]\right|_{x=2}$ -10

31. $\left.\frac{d}{dx}(3u - 2v + 2uv)\right|_{x=2}$ -12

In exercises 32 – 37, use the following table to answer the questions.

x	$F(x)$	$F'(x)$	$F''(x)$	$G(x)$	$G'(x)$	$G''(x)$
3	5	4	−3	2	7	−2
5	8	6	10	−6	−4	11

32. If $H(x) = (F(x))^2$, then what is $H'(3)$? 40

33. If $H(x) = \frac{F(x)}{G(x)}$, then what is $H'(3)$? $-\frac{27}{4}$

34. If $H(x) = F(x) \cdot G(x)$, then what is $H''(3)$? 40

35. If $H(x) = G(F(x))$, then what is $H'(3)$? -16

36. If $H(x) = G(F(x))$, then what is $H''(3)$? 96

37. If $H(x) = \ln(F(x))$, then what is $H'(3)$? $\frac{4}{5}$

In exercises 38 – 43, use the function $f(x) = x^3$ to answer the questions.

38. Find $f'(x)$. $3x^2$

39. Evaluate $f'(2)$. Explain the meaning of this. 12 the instantaneous rate of change at $x=2$ is 12 ; the slope of the tangent at $x = 2$ is 12

40. Find the average rate of change over the interval $[-2,2]$ 4

In exercises 44 and 45, use the table to answer the questions.

x	-5	0	5	10
$f(x)$	1	5	4	-8

44. Using the values in the table, estimate $f'(3)$.

45. Use the data in the table to estimate the value of $f'(5)$

46. Explain why there must be a value c, $5 < x < 10$, such that $f(x) = 0$.

In exercises 47 – 64, find the derivative.

47. $f(x) = \frac{-x^4}{2} + \frac{4}{x^3} - 3x + 4$

48. $f(x) = 2x - \frac{2x^2}{\sqrt{4x}} + \sqrt[5]{x^2}$

49. $f(x) = 2\sin x \cos x + \sin x$ 50. $g(x) = \frac{3 - 2x - x^2}{x^2 - 1}$

51. $f(x) = x^2 \sin(4x)$

52. $h(x) = \frac{\cos(7x)}{4x}$

53. $f(x) = \sqrt{1 - x^3}$

54. $y = 2\cos^4(3x + 1)$

55. $y = x^2(x^2 - 3)^3$

56. $y = \csc^3(2x^2)$

57. $f(x) = x^3 \tan^2 x$

58. $f(x) = \ln(1 + e^{3x})$

59. $y = \ln(x^2 + 3)$

60. $y = e^{x^2 - 3x}$

61. $g(t) = \tan^{-1}(3t - 4)$

62. $h(x) = x \sin^{-1} x$

63. $f(x) = \arctan(e^x)$

64. $y = \arcsin(5x)$

47) $f'(x) = -2x^3 - \frac{12}{x^4} - 3$

48) $f'(x) = 2 - \frac{3}{2\sqrt{x}} + \frac{2}{5\sqrt[5]{x^3}}$

49) $f'(x) = 2(\cos^2 x - \sin^2 x) + \cos x$

50) $g'(x) = \frac{-2x^2 + 2}{(x^2-1)^2}$

51) $f'(x) = 4x^2\cos(4x) + 2x\sin(4x)$

52) $h'(x) = \frac{7x\sin 7x - \cos 7x}{4x^2}$

53) $f'(x) = \frac{3x^2}{2}(1-x^3)$

54) $y' = -24\cos^3(3x+1)\sin(3x+1)$

55) $y' = (x^2-3)^2(8x^3-2x)$

56) $y' = -12\csc^3(2x^2)\cot(2x^2)$

57) $f'(x) = 2x^3\tan x(\sec x)^2 + 3x^2\tan^2 x$

58) $f'(x) = \frac{3e^{3x}}{1+e^{3x}}$

59) $y' = \frac{2x}{x^2+3}$

60) $y' = (2x-3)e^{x^2-3x}$

61) $g'(t) = \frac{1}{t-1}$

62) $h'(x) = \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$

63) $f'(x) = \frac{e^x}{1+e^x}$

64) $y' = \frac{5}{\sqrt{1-25x^2}}$