

4.6 pg. 251 2-24 even

2)  $S = 4\pi r^2$

$$\frac{ds}{dt} = 8\pi r \frac{dr}{dt}$$

4)  $P = RI^2$

a)  $\frac{dP}{dt} = R(2I) \frac{dI}{dt} + I^2 \frac{dR}{dt}$   
 $= 2RI \frac{dI}{dt} + I^2 \frac{dR}{dt}$

b)  $\frac{dP}{dt} = 0$

$$0 = 2RI \frac{dI}{dt} + I^2 \frac{dR}{dt}$$

6)  $A = \frac{1}{2} ab \sin \theta$

$$\frac{dA}{dt} = \frac{1}{2} \left( \frac{da}{dt} b \sin \theta + a \frac{db}{dt} \sin \theta + ab \cos \theta \frac{d\theta}{dt} \right)$$

8)  $r = \text{plate radius}$   $\frac{dr}{dt} = 0.01$

$$A = \pi r^2 \quad \frac{dA}{dt} = ? \quad r = 50$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi(50)(0.01)$$

$$\frac{dA}{dt} = \pi \text{ cm}^2/\text{sec}$$

b)  $\frac{dx}{dt} = 1$   $\frac{dy}{dt} = -2$   $\frac{dz}{dt} = 1$   $x = 4, y = 3, z = 2$

a) volume =  $xyz$

$$\frac{dV}{dt} = \frac{dx}{dt} yz + x \frac{dy}{dt} z + xy \frac{dz}{dt}$$

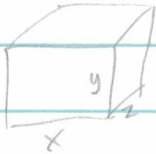
$$= 1(3)(2) + 4(-2)(2) + 4(3)(1) = 6 - 16 + 12 = 2 \text{ m}^3/\text{sec}$$

b)  $SA = 2(xy + xz + yz)$

$$\frac{dSA}{dt} = 2 \left( x \frac{dy}{dt} + \frac{dx}{dt} y + \frac{dx}{dt} z + x \frac{dz}{dt} + \frac{dy}{dt} z + y \frac{dz}{dt} \right)$$

$$= 2(4(-2) + 1(3) + 1(2) + 4(1) + (-2)(2) + 3(1))$$

$$= 2(-8 + 3 + 2 + 4 - 4 + 3) = 0 \text{ m}^2/\text{sec}$$



$$c) s = \sqrt{x^2 + y^2 + z^2}$$

$$\frac{ds}{dt} = \frac{1}{2}(x^2 + y^2 + z^2)^{-\frac{1}{2}}(2x \frac{dx}{dt} + 2y \frac{dy}{dt} + 2z \frac{dz}{dt})$$

$$= \frac{1}{2}(4^2 + 3^2 + 2^2)^{-\frac{1}{2}}(2(4)(1) + 2(3)(-2) + 2(2)(1))$$

$$= \frac{1}{2\sqrt{29}}(0) = 0 \text{ m/sec}$$

12)  $r = \text{radius}$

$$S = 4\pi r^2$$

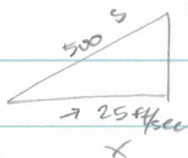
$$V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = kS$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$kS = \underbrace{4\pi r^2}_{S} \frac{dr}{dt}$$

$$k = \frac{dr}{dt}$$

14)



$$\frac{ds}{dt} = ? \quad h = 300$$

$$s = 500 \quad \frac{dx}{dt} = 25 \text{ ft/sec}$$

$$x^2 + 300^2 = s^2$$

$$x^2 + 300^2 = 500^2$$

$$x^2 + 90000 = s^2$$

$$x^2 = 500^2 - 300^2$$

$$2x \frac{dx}{dt} + 0 = 2s \frac{ds}{dt}$$

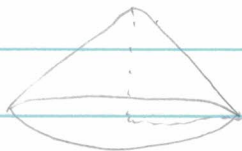
$$x = 400$$

$$2(400)(25) + 0 = 2(500) \frac{ds}{dt}$$

$$20000 = 1000 \frac{ds}{dt}$$

$$\frac{ds}{dt} = 20 \text{ ft/sec}$$

(6)



$$\frac{dV}{dt} = 10$$

$$h = 4$$

$$h = \frac{3}{8}d \rightarrow \frac{3}{8}(2r) \rightarrow \frac{6}{8}r \rightarrow r = \frac{4}{3}h$$

$$\frac{dh}{dt} = ? \quad \frac{dr}{dt} = ?$$

$$r = \frac{4}{3}(4)$$

$$r = \frac{16}{3}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{4}{3}h\right)^2 h$$

$$V = \pi \frac{16}{27} h^3$$

$$\frac{dV}{dt} = \frac{16}{27}\pi (3) h^2 \frac{dh}{dt}$$

$$10 = \frac{16}{9}\pi (4)^2 \frac{dh}{dt}$$

$$10 = \frac{256}{9}\pi \frac{dh}{dt}$$

$$\frac{9}{256\pi} 10 = \frac{dh}{dt}$$

$$\frac{45}{128\pi} \text{ m/min} \Rightarrow \frac{45}{128\pi}(100) = \frac{1125}{32\pi} \text{ cm/min} \approx 11.19$$

$$b) r = \frac{4}{3}h$$

$$\frac{dr}{dt} = \frac{4}{3} \frac{dh}{dt}$$

$$= \frac{4}{3} \left( \frac{1125}{32\pi} \right)$$

$$= \frac{375}{8\pi} \text{ cm/min} \approx 14.92$$

$$18) V = \frac{\pi}{3} y^2 (3R - y) \quad \frac{dV}{dt} = -6 \text{ m}^3/\text{min} \quad y = 8 \quad R = 13$$

$$a) \frac{dy}{dt} = ?$$

$$V = \frac{\pi}{3} y^2 (3(13) - y) = \frac{\pi}{3} y^2 (39 - y) = 13\pi y^2 - \frac{\pi}{3} y^3$$

$$\frac{dV}{dt} = 26\pi y \frac{dy}{dt} - \pi y^2 \frac{dy}{dt}$$

$$-6 = 26\pi (8) \frac{dy}{dt} - \pi (8)^2 \frac{dy}{dt}$$

$$-6 = 208\pi \frac{dy}{dt} - 64\pi \frac{dy}{dt}$$

$$-6 = 144\pi \frac{dy}{dt}$$

$$\frac{-6}{144\pi} = \frac{dy}{dt} \rightarrow \frac{dy}{dt} = \frac{-1}{24\pi}$$

$$b) r^2 + (13 - y)^2 = 13^2$$

$$r^2 = 13^2 - (13 - y)^2$$

$$(13 - y)(13 - y)$$

$$r = \sqrt{169 - (169 - 26y + y^2)}$$

$$r = \sqrt{26y - y^2}$$

$$c) \frac{dr}{dt} = \frac{1}{2} (26y - y^2)^{-1/2} (26 \frac{dy}{dt} - 2y \frac{dy}{dt})$$

$$= \frac{1}{2} (26(8) - 8^2)^{-1/2} (26 \left(\frac{-1}{24\pi}\right) - 2(8) \left(\frac{-1}{24\pi}\right))$$

$$= \frac{1}{2} \left(\frac{1}{12}\right) \left(-\frac{26}{24\pi} + \frac{2}{3\pi}\right) = \frac{-5}{288\pi}$$

$$20) \frac{dV}{dt} = 2.5 \text{ ft}^3/\text{min} \quad h = 2 \text{ ft}$$

$$V = Bh = \frac{1}{2} \left( \frac{4}{3}h \right) (h) (15)$$

$$= 10h^2$$

$$\frac{dV}{dt} = 20h \frac{dh}{dt}$$

$$2.5 = 20(2) \frac{dh}{dt}$$

$$2.5 = 40 \frac{dh}{dt}$$

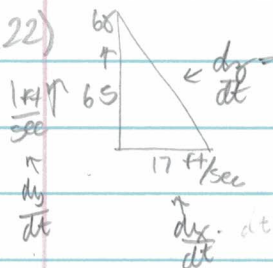
$$\frac{dh}{dt} = \frac{1}{16} \text{ ft}/\text{min} \approx 0.0625$$



$$\frac{4}{3} = \frac{x}{h}$$

$$x = \frac{4}{3}h$$

22)



$$x^2 + y^2 = z^2$$

$$17(3) = 51 \text{ ft} \leftarrow x$$

$$65 + 3 = 68 \text{ ft} \leftarrow y$$

$$68^2 + 51^2 = z^2$$

$$7225 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt} \quad z = 85$$

$$2(51)(17) + 2(68)(1) = 2(85) \frac{dz}{dt}$$

$$1734 + 136 = 170 \frac{dz}{dt}$$

$$1870 = 170 \frac{dz}{dt}$$

$$\frac{dz}{dt} = 11 \text{ ft}/\text{sec}$$

$$y = x^3 - 4x$$

$$24) \frac{dy}{dt} = 3x^2 \frac{dx}{dt} - 4 \frac{dx}{dt}$$

$$a) \frac{dy}{dt} = 3(-3)^2(-2) - 4(-2) = -54$$

$$b) \frac{dy}{dt} = 3(1)^2(-2) - 4(1) = -10$$

$$c) \frac{dy}{dt} = 3(4)^2(-2) - 4(4) =$$