

# Related Rates Worksheet

1)  $C = 2\pi r$        $A = \pi r^2$        $\frac{dA}{dt} = 2 \frac{dC}{dt}$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

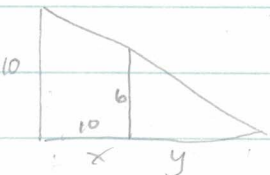
$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$2 \frac{dC}{dt} = 2\pi r \frac{dr}{dt}$$

$$2(2\pi \frac{dr}{dt}) = 2\pi r \frac{dr}{dt}$$

$$4\pi \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$$

$$2 = r \quad \text{(B)}$$

2) 

$$\frac{dx}{dt} = 4$$

$$\frac{10}{x+y} = \frac{6}{y}$$

$$10y = 6(x+y)$$

$$10y = 6x + 6y$$

$$4y = 6x$$

$$y = \frac{3}{2}x$$

Find  $\frac{d(x+y)}{dt}$

$$\Rightarrow \frac{dx}{dt} + \frac{dy}{dt}$$

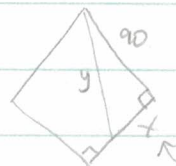
$$\frac{dy}{dt} = \frac{3}{2} \frac{dx}{dt}$$

$$= \frac{3}{2}(4)$$

$$\frac{dy}{dt} = 6$$

$$\frac{d(x+y)}{dt} = 4+6$$

$$\frac{d(x+y)}{dt} = 10 \quad \text{(B)}$$

3) 

$$\frac{dx}{dt} = 25$$

$$x = \frac{1}{3}(90) = 30$$

$$x^2 + 90^2 = y^2$$

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$2(30)(25) = 2\sqrt{9000} \frac{dy}{dt}$$

$$\frac{1500}{2\sqrt{9000}} = \frac{dy}{dt}$$

$$\frac{1500}{2 \cdot 30\sqrt{10}} = \frac{dy}{dt}$$

$$\frac{25}{\sqrt{10}} = \frac{dy}{dt}$$

$$\frac{25 \cdot \sqrt{10}}{10} = \frac{5}{2}\sqrt{10} \quad \text{(A)}$$

$$30^2 + 90^2 = y^2$$

$$900 + 8100 = y^2$$

$$9000 = y^2$$

$$y = \sqrt{9000}$$

4)  $x = \frac{1}{11}$     $y = \frac{1}{11}$     $\frac{dy}{dt} = 5$     $\frac{dx}{dt} = ?$

$$5x^3 + 6y^3 = xy$$

$$15x^2 \frac{dx}{dt} + 18y^2 \frac{dy}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt}$$

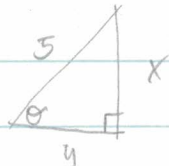
$$15\left(\frac{1}{11}\right)^2 \frac{dx}{dt} + 18\left(\frac{1}{11}\right)^2 (5) = \left(\frac{1}{11}\right)(5) + \left(\frac{1}{11}\right) \frac{dx}{dt}$$

$$\frac{15}{121} \frac{dx}{dt} + \frac{90}{121} = \frac{5}{11} + \frac{1}{11} \frac{dx}{dt}$$

$$\left(\frac{15}{121} - \frac{1}{11}\right) \frac{dx}{dt} = \frac{5}{11} - \frac{90}{121}$$

$$\frac{4}{121} \frac{dx}{dt} = \frac{-35}{121}$$

$$\frac{dx}{dt} = \frac{-35}{4} \quad \text{(F)}$$

5)   $\frac{d\theta}{dt} = 2$      $\frac{dx}{dt} = ?$   
 $x = 4$

$$4^2 + y^2 = 5^2$$

$$\sin \theta = \frac{x}{5}$$

$$16 + y^2 = 25$$

$$5 \sin \theta = x$$

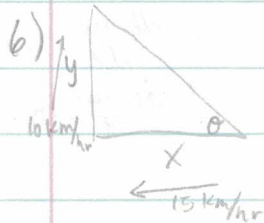
$$y^2 = 9$$

$$5 \cos \theta \frac{d\theta}{dt} = \frac{dx}{dt}$$

$$y = 3$$

$$5 \left(\frac{3}{5}\right) (2) = \frac{dx}{dt}$$

$$6 = \frac{dx}{dt} \quad \text{(D)}$$



$$\frac{dx}{dt} = -15$$

$$x = 4$$

$$\frac{dy}{dt} = 10$$

$$y = 3$$

a)  $x^2 + y^2 = z^2$      $\rightarrow$  b)  $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$   
 $4^2 + 3^2 = z^2$      $2(4)(-15) + 2(3)(10) = 2(5) \frac{dz}{dt}$   
 $16 + 9 = z^2$      $-120 + 60 = 10 \frac{dz}{dt}$   
 $25 = z^2$      $-\frac{60}{10} = \frac{dz}{dt}$   
 $z = 5$      $\frac{dz}{dt} = -6$

c)  $\tan \theta = \frac{y}{x}$   
 $\sec^2 \theta \frac{d\theta}{dt} = x \frac{dy}{dt} - y \frac{dx}{dt}$   
 $\sec^2 \theta \frac{d\theta}{dt} = \frac{4(10) - 3(-15)}{4^2}$   
 $\left(\frac{5}{4}\right)^2 \frac{d\theta}{dt} = \frac{85}{16}$   
 $\frac{25}{16} \frac{d\theta}{dt} = \frac{85}{16}$   
 $\frac{d\theta}{dt} = \frac{85}{16} \left(\frac{16}{25}\right)$   
 $\frac{d\theta}{dt} = \frac{17}{5} \text{ rad/hr}$

7) a)  $w=3$   $(3, \frac{1}{3^2}) \rightarrow (3, \frac{1}{9})$

$x^{-2}$

$$y' = -2x^{-3}$$

$$y' = -\frac{2}{3^3}$$

$$y' = -\frac{2}{27}$$

$$y = -\frac{2}{27}(x-3) + \frac{1}{9}$$

$$= -\frac{2}{27}x + \frac{6}{27} + \frac{1}{9}$$

$$y = -\frac{2}{27}x + \frac{3}{9}$$

$$0 = -\frac{2}{27}x + \frac{1}{3}$$

$$-\frac{1}{3} = -\frac{2}{27}x$$

$$x = \left(-\frac{1}{3}\right) \left(-\frac{27}{2}\right)$$

$$x = \frac{9}{2}$$

$$\boxed{k = \frac{9}{2}}$$

b)  $y' = -2x^{-3}$

$$y' = -\frac{2}{w^3}$$

$$y = -\frac{2}{w^3}(x-w) + \frac{1}{w^2}$$

$$y = -\frac{2}{w^3}x + \frac{2}{w^2} + \frac{1}{w^2}$$

$$0 = -\frac{2}{w^3}x + \frac{3}{w^2}$$

$$\frac{2}{w^3}x = \frac{3}{w^2}$$

$$x = \frac{3}{w^2} \left(\frac{w^3}{2}\right)$$

$$\boxed{k = \frac{3w}{2}}$$

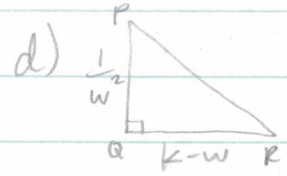
c)  $\frac{dw}{dt} = 7$   $w=5$

$$k = \frac{3w}{2}$$

$$\frac{dk}{dt} = \frac{3}{2} \frac{dw}{dt}$$

$$= \frac{3}{2}(7)$$

$$\boxed{\frac{dk}{dt} = \frac{21}{2}}$$



$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(k-w)\left(\frac{1}{w^2}\right)$$

$$= \frac{1}{2}\left(\frac{3}{2}w - w\right)\left(\frac{1}{w^2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{2}w\right)\left(\frac{1}{w^2}\right)$$

$$= \frac{1}{4} \frac{w}{w^2}$$

$$= \frac{1}{4w}$$

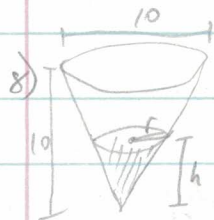
$$\frac{dA}{dt} = -\frac{1}{4}w^{-2} \frac{dw}{dt}$$

$$= -\frac{1}{4}(5)^{-2} \frac{dw}{dt}$$

$$= -\frac{1}{100}(7)$$

$$= -\frac{7}{100} < 0$$

decreasing



$$\frac{dh}{dt} = -\frac{3}{10}$$

$$\frac{h}{r} = \frac{10}{5}$$

$$h = 2r \rightarrow r = \frac{h}{2}$$

a)  $V = \frac{1}{3}\pi r^2 h$      $h = 5$      $r = \frac{5}{2}$

$$V = \frac{1}{3}\pi \left(\frac{5}{2}\right)^2 (5)$$

$$= \frac{1}{3}\pi \left(\frac{25}{4}\right)(5)$$

$$V = \frac{125}{12}\pi \text{ cm}^3$$

b)  $V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$$V = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{12}\pi (3h^2) \frac{dh}{dt}$$

$$= \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$= \frac{1}{4}\pi (5)^2 \left(-\frac{3}{10}\right)$$

$$= -\frac{75\pi}{40}$$

$$\frac{dV}{dt} = -\frac{15\pi}{8} \text{ cm}^3/\text{hr}$$

c)  $\frac{dV}{dt} = k\pi r^2$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi r^2 (2r)$$

$$= \frac{2}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{2}{3}\pi 3r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$$

$$= 2\pi r^2 \left(\frac{1}{2} \frac{dh}{dt}\right)$$

$$\frac{dV}{dt} = \pi r^2 \left(-\frac{3}{10}\right)$$

$$k\pi r^2 = \pi r^2 \left(-\frac{3}{10}\right)$$

$$k = -\frac{3}{10}$$

from b)

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$= \frac{1}{4}\pi h^2 \left(-\frac{3}{10}\right)$$

$$= -\frac{3}{40}\pi h^2$$

$$= -\frac{3}{40}\pi (2r)^2$$

$$= -\frac{3}{40}\pi (4r^2)$$

$$\frac{dV}{dt} = -\frac{3}{10}\pi r^2$$

$$k\pi r^2 = -\frac{3}{10}\pi r^2$$

$$k = -\frac{3}{10}$$