CONCEPTS:

1. When looking for absolute extrema, where do the possible extrema exist, and how do you find them?
   - Extrema may exist at endpoints and critical points (\( f' = 0 \) or undefined)
   - The max/min values can be obtained by plugging in the endpoints
2. How do you justify relative extrema?
   - Critical points and critical points into the function
   - \( f' \) changes signs (positive to negative or negative to positive)
3. How do you justify that a function is increasing or decreasing?
   - Increasing: \( f' > 0 \)
   - Decreasing: \( f' < 0 \)
4. How do you justify that a function is concave up or concave down?
   - Concave up: \( f'' > 0 \)
   - Concave down: \( f'' < 0 \)
5. How do you justify that a function has a point of inflection?
   - \( f'' \) changes signs
6. Using the graph of \( g(x) \) below, determine the signs of \( g'(x) \) and \( g''(x) \) at each point. Explain your reasoning.

<table>
<thead>
<tr>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( g'(x) )</td>
<td>reason</td>
<td>( g''(x) )</td>
</tr>
<tr>
<td>( a )</td>
<td>-</td>
<td>( g(x) ) is decreasing</td>
<td>+</td>
</tr>
<tr>
<td>( b )</td>
<td>0</td>
<td>horizontal tangent</td>
<td>+</td>
</tr>
<tr>
<td>( c )</td>
<td>0</td>
<td>horizontal tangent</td>
<td>-</td>
</tr>
<tr>
<td>( d )</td>
<td>-</td>
<td>( g(x) ) is decreasing</td>
<td>-</td>
</tr>
</tbody>
</table>
ABCALC Apps of Derivatives Review Session Problems

7. Given the graph of $f'$ below answer each of the following questions, and justify your response with a statement that contains the phrase “since $f'$ _______ ...”

a) When is $f$ increasing?
   (a,b) and (d,e) since $f' > 0$

b) When is $f$ decreasing?
   (b,d) since $f' < 0$

c) When is $f$ concave up?
   (c,e) since $f'$ is increasing

d) When is $f$ concave down?
   (a,c) since $f'$ is decreasing

e) When does $f$ have a relative maximum?
   b since $f'$ changes from $+$ to $-$

f) When does $f$ have a relative minimum?
   d since $f'$ changes from $-$ to $+$

g) When does $f$ have a point of inflection?
   c since $f''$ changes sign

Find the value of $c$ guaranteed by the MVT for $f(x) = 4x^2 + 5x$ on the interval $[-2, 1]$.

$$f'(x) = \frac{f(1) - f(-2)}{1 - (-2)}$$

$$8x + 5 = \frac{[4(1)^2 + 5(1)] - [4(-2)^2 + 5(-2)]}{3}$$

$$8x + 5 = \frac{9 - 4}{3}$$

$$8x + 5 = 1$$

$$8x = -4$$

$$x = -\frac{4}{8} \Rightarrow c = -\frac{1}{2}$$
Suppose \( y = x^3 - 3x \). [No Calculator]

a) Find the zeros of the function.
\[
y = x(x^2 - 3)
\]
x = 0, \( x^2 - 3 = 0 \)
x = \pm \sqrt{3}
\( 0, -\sqrt{3}, \sqrt{3} \)

b) Determine where \( y \) is increasing or decreasing and justify your response.
\[
y' = 3x^2 - 3 \quad 0 = 3x^2 - 3 \quad (x + 1)(x - 1) = 0
\]
inc: \((\infty, -1)\) and \((1, \infty)\) since \( f' > 0 \)
dec: \((-1, 1)\) since \( f' < 0 \)

c) Determine all local extrema and justify your response.
min: \((-1, 2)\) since \( f' \) changes from - to +
max: \((-1, 2)\) since \( f' \) changes from + to -

d) Determine the points where \( y \) is concave up or concave down, and find any points of inflection.
Justify your responses.
\[
y'' = 6x
\]
concave down: \((-\infty, 0)\) since \( f'' < 0 \)
concave up: \((0, \infty)\) since \( f'' > 0 \)

e) Use all your information to sketch a graph of this function.

The function \( f \) is continuous on \([0, 3]\) and satisfies the following:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0 &lt; ( x &lt; 1 )</th>
<th>1</th>
<th>1 &lt; ( x &lt; 2 )</th>
<th>2</th>
<th>2 &lt; ( x &lt; 3 )</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>0</td>
<td>Neg</td>
<td>-2</td>
<td>Neg</td>
<td>0</td>
<td>Pos</td>
<td>3</td>
</tr>
<tr>
<td>( f' )</td>
<td>-3</td>
<td>Neg</td>
<td>0</td>
<td>Pos</td>
<td>DNE</td>
<td>Pos</td>
<td>4</td>
</tr>
<tr>
<td>( f'' )</td>
<td>0</td>
<td>Pos</td>
<td>1</td>
<td>Pos</td>
<td>DNE</td>
<td>Pos</td>
<td>0</td>
</tr>
</tbody>
</table>

a) Find the absolute extrema of \( f \) and where they occur.
\[
f' \quad \frac{-}{+} \quad \frac{+}{+} \quad \text{abs. max.} \quad (3, 3)
\]
\[
f'' \quad \text{min} \quad \quad \quad \text{abs. min.} \quad (1, -2)
\]

b) Find any points of inflection.
\[\text{none. since } f'' \text{ does not change signs}\]

c) Sketch a possible graph of \( f \).