ABCALC Apps of Derivatives Review Session Problems

CONCEPTS:

1. When looking for absolute extrema, where do the possible extrema exist, and how do you find them?

extremas may exist at endpoints and critical points ($f'=0$ or $f'=$ undefined)
the max/min values can be obtained by plugging in the endpoints and critical points into the function

2. How do you justify relative extrema?

f' changes signs (positive to negative or negative to positive)

3. How do you justify that a function is increasing or decreasing?

increasing: $f'>0$
decreasing: $f'<0$

4. How do you justify that a function is concave up or concave down?

concave up: $f''>0$
concave down: $f''<0$

5. How do you justify that a function has a point of inflection?

f'' changes signs

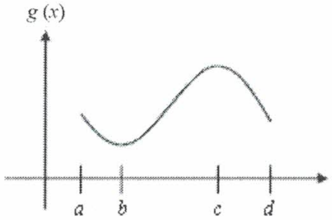
6. Using the graph of $g(x)$ below, determine the signs of $g'(x)$ and $g''(x)$ at each point. Explain your reasoning.

At $x = a$ ...

At $x = b$ ...

At $x = c$ ...

At $x = d$ ...



x	$g'(x)$	reason	$g''(x)$	reason
a	−	g(x) is decreasing	+	g(x) is concave up
b	0	horizontal tangent local min	+	g(x) is concave up
c	0	horizontal tangent local max	−	g(x) is concave down
d	−	g(x) is decreasing	−	g(x) is concave down

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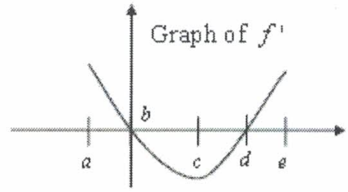
7. Given the graph of f' below answer each of the following questions, and justify your response with a statement that contains the phrase "since f' ________________ ..."



a) When is f increasing?

(a,b) and (d,e) since $f'>0$

b) When is f decreasing?

(b,d) since $f'<0$

c) When is f concave up?

(c,e) since f' is increasing

d) When is f concave down?

(a,c) since f' is decreasing

e) When does f have a relative maximum?

b since f' changes from + to −

f) When does f have a relative minimum?

d since f' changes from − to +

g) When does f have a point of inflection?

c since f'' changes sign

Find the value of c guaranteed by the MVT for $f(x)=4x^2+5x$ on the interval [−2, 1].

$$f'(x)=\frac{f(1)-f(-2)}{1--2}$$

$$8x+5=\frac{[4(1)^2+5(1)]-[4(-2)^2+5(-2)]}{3}$$

$$8x+5=\frac{9-6}{3}$$

$$8x+5=1$$

$$8x=-4$$

$$x=\frac{-4}{8} \Rightarrow \boxed{c=\frac{-1}{2}}$$

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Suppose $y = x^3 - 3x$. [No Calculator]

a) Find the zeros of the function.

$y = x^3 - 3x$ $\quad x = 0, \; x^2 - 3 = 0$ $\quad \{0, -\sqrt{3}, \sqrt{3}\}$

$y = x(x^2-3)$ $\quad x^2 = 3 \rightarrow x = \pm\sqrt{3}$

b) Determine where y is increasing or decreasing and justify your response.

$y' = 3x^2 - 3$ $\quad x^2 - 1 = 0$ $\quad y' \; + \; - \; +$ $\quad$ inc: $(-\infty, -1)$ and $(1, \infty)$ since $f' > 0$

$0 = 3x^2 - 3$ $\quad (x+1)(x-1) = 0$ $\quad \nearrow -1 \searrow 1 \nearrow$ $\quad$ dec: $(-1, 1)$ since $f' < 0$

$0 = 3(x^2 - 1)$ $\quad x = -1, x = 1$

c) Determine all local extrema and justify your response.

min: $(1, -2)$ since f' changes from $-$ to $+$

max: $(-1, 2)$ since f' changes from $+$ to $-$

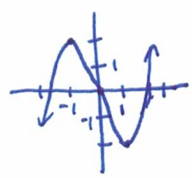
d) Determine the points where y is concave up or concave down, and find any points of inflection. Justify your responses.

$y'' = 6x$ $\quad - \; + \;$ (0) $\quad$ concave down: $(-\infty, 0)$ since $f'' < 0$

$0 = 6x$ $\quad$ concave up: $(0, \infty)$ since $f'' > 0$

$x = 0$

e) Use all your information to sketch a graph of this function.



The function f is continuous on [0, 3] and satisfies the following:

	(0,0)	$\searrow$	(1,-2)	$\nearrow$	(2,0)	$\nearrow$	(3,3)
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3
f	0	Neg	–2	Neg	0	Pos	3
f'	–3	Neg	0	Pos	DNE	Pos	4
f''	0	Pos	1	Pos	DNE	Pos	0

a) Find the absolute extrema of f and where they occur.

f' $\; - \; + \; +$ $\quad$ abs. max. $(3, 3)$

$\searrow 1 \nearrow 2 \nearrow$ $\quad$ abs. min. $(1, -2)$

min

b) Find any points of inflection.

none. since f'' does not change signs

c) Sketch a possible graph of f.