3. Find all local extrema and justify your response for each function:
   a) \( y = -2x^4 + 6x^2 - 3 \)
   \[ \text{min: (0, -3)} \]
   \[ \text{max: (2, 6)} \]
   b) \( y = xe^x \)
   \[ \text{min: (1, e)} \]

4. Determine the intervals on which the graph of each function is concave up or concave down and determine all points of inflection. Justify your responses:
   a) \( y = \frac{1}{3}x^4 + \frac{1}{2}x^2 - \frac{2}{3}x^3 - \frac{1}{2}x^2 + x - 4 \)
   \[ \text{Concave up: (3, \infty)} \]
   \[ \text{Concave down: (-\infty, 3)} \]
   \[ \text{Point of inflection: (3, -130.5)} \]
   b) \( y = 2x^3 + 3 \)
   \[ \text{Concave up: (-\infty, 0)} \]
   \[ \text{Concave down: (0, \infty)} \]
   \[ \text{Point of inflection: (0, 0)} \]

5. If \( f \) is continuous on [0, 3] and satisfies the following:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>0 &lt; x &lt; 1</th>
<th>1</th>
<th>1 &lt; x &lt; 2</th>
<th>2</th>
<th>2 &lt; x &lt; 3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>+</td>
<td>2</td>
<td>+</td>
<td>0</td>
<td>--</td>
<td>-2</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>3</td>
<td>+</td>
<td>0</td>
<td>--</td>
<td>DNE</td>
<td>--</td>
<td>-3</td>
</tr>
<tr>
<td>( f''(x) )</td>
<td>0</td>
<td>--</td>
<td>-1</td>
<td>--</td>
<td>DNE</td>
<td>--</td>
<td>0</td>
</tr>
</tbody>
</table>

   a) Find the absolute extrema of \( f \) and where they occur. Justify your response.
   \[ \text{Max: (1, 2) since } f' \text{ changes signs at (1, 2)} \]

   b) Find any points of inflection. Justify your response.
   \[ \text{None, there are no points where } f'' \text{ changes signs} \]

   c) Sketch a possible graph of \( f \).
6. Let $f$ be a function that is continuous on the interval $[0, 4]$. The function $f$ is twice differentiable except at $x = 2$. The function $f$ and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of $f$ do not exist at $x = 2$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$0 &lt; x &lt; 1$</th>
<th>$1 &lt; x &lt; 2$</th>
<th>$2 &lt; x &lt; 3$</th>
<th>$3 &lt; x &lt; 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>$-1$</td>
<td>Negative</td>
<td>0</td>
<td>Positive</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>Positive</td>
<td>0</td>
<td>Positive</td>
<td>DNE</td>
</tr>
<tr>
<td>$f''(x)$</td>
<td>Negative</td>
<td>0</td>
<td>Positive</td>
<td>DNE</td>
</tr>
</tbody>
</table>

a) Describe the behavior of $f(x)$ in each interval using the information above.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$0 &lt; x &lt; 1$</th>
<th>$1 &lt; x &lt; 2$</th>
<th>$2 &lt; x &lt; 3$</th>
<th>$3 &lt; x &lt; 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>inc.</td>
<td>conc. down</td>
<td>inc.</td>
<td>conc. up</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>dec.</td>
<td>conc. down</td>
<td>dec.</td>
<td>conc. up</td>
</tr>
</tbody>
</table>

b) For $0 < x < 4$, find all values of $x$ at which $f$ has a relative extremum. Determine whether $f$ has a relative maximum or a relative minimum at each of these values. Justify your answer.

max: $(2, 2)$ since $f'$ changes from positive to negative at $(2, 2)$

c) On the axes provided, sketch the graph of a function that has all the characteristics of $f$.

Use the graph of $f'(x)$ defined on $[0, 6]$ provided below to estimate the following:

a) When is $f$ increasing? When is $f$ decreasing? Justify your response.

inc. $(0, 1); (3, 5)$ since $f'>0$ 
dec. $(1, 3); (5, 6)$ since $f'<0$

b) Determine the $x$-coordinates of all local extrema. Justify your response.

$x = 1, x = 3, x = 5$ since $f'(x) = 0$

c) Determine when $f$ is concave up and concave down. Justify your response.

conc. up. $(2, 4)$ since $f''$ is increasing 
conc. down. $(0, 2); (4, 6)$ since $f''$ is decreasing

d) Determine whether $f$ has any points of inflection. Justify your response.

$x = 2; x = 4$ since $f''$ would change signs at these values

9. [No Calculator Allowed] Let $f$ be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of $f''$, the derivative of $f$, consists of one line segment and a semicircle, as shown below.

a) On what intervals, if any, is $f$ increasing? Decreasing? Justify your answer.

inc. $(-3, -2)$ since $f'>0$ 
dec. $(-3, -4)$ since $f'<0$

b) Find all values of $x$ for which $f$ assumes a relative maximum. Justify your answer.

$x = -2$ since $f'$ changes from positive to negative

c) Where is the graph of $f$ concave up? concave down? Justify your answer.

conc. up. $(0, 2)$ since $f''$ is increasing 
conc. down. $(-3, -2); (2, 4)$ since $f''$ is decreasing

d) Find the $x$-coordinate of each point of inflection of the graph of $f$ on the open interval $-3 < x < 4$. Justify your answer.

$x = 0, x = 2$ since $f''$ changes from decreasing to increasing at $x = 0$ and inc. to dec. at $x = 2$

e) Find an equation for the line tangent to the graph of $f$ at the point $(0, 3)$.

$y = -2x + 3$

f) Sketch a possible graph of $f$.

7. [Calculator Required] Let $f$ be a function defined for $x \geq 0$ with $f(0) = 5$ and $f'$, the first derivative of $f$, given by $f'(x) = e^{-x} \sin(x^2)$. The graph of $y = f'(x)$ is shown below.

a) Use the graph of $f'$ to determine whether the graph of $f$ is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

concave down since $f'$ is decreasing on $1.7 < x < 1.9$

b) Write an equation for the line tangent to the graph of $f$ at the point $(2, 5.623)$.

$y = -0.459x + 6.541$