AP Calculus
4.3 Worksheet

All work must be shown in this course for full credit. Unsupported answers may receive NO credit.

1. Complete each statement with the correct word.

a) When f' is positive, the graph of f is increasing,.

b) When f' is negative, the graph of f is decreasing,.

c) When f'' is positive, the graph of f is concave upward.

d) When f'' is negative, the graph of f is concave downward.

e) When f' is increasing, the graph of f is concave upward.

f) When f' is decreasing, the graph of f is concave downward.

2. Use the function $y = 3x - x^3 + 5$. [No calculator allowed]

a) Where is the function increasing? Justify your response.

$(-1,1)$ since y' is positive on $(-1,1)$

b) Where is the function decreasing? Justify your response.

$(-\infty,-1)$ & $(1,\infty)$ since y' is negative on $(-\infty,-1)$ & $(1,\infty)$

c) Where is the function concave up? Justify your response.

$(-\infty,0)$ since y'' is positive on $(-\infty,0)$

d) Where is the function concave down? Justify your response.

$(0,\infty)$ since $y'' < 0$ on $(0,\infty)$

e) Where are the point(s) of inflection? Justify your response.

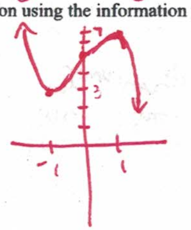
$(0,5)$ since y'' changes sign on $(0,5)$

f) Find ALL extrema and justify your response.

$(-1,3)$ min y' changes from negative to positive

$(1,7)$ max y' changes from positive to negative

g) Create a *sketch* of the function using the information you have found from *a – f*.



3. Find all local extrema and justify your response for each function:

a) $y = -2x^3 + 6x^2 - 3$

min: $(0,-3)$

max: $(2,5)$

b) $y = xe^{1/x}$

min: $(1,e)$

4. Determine the intervals on which the graph of each function is concave up or concave down and determine all points of inflection. Justify your responses.

a) $y = \frac{1}{20}x^5 + \frac{1}{4}x^4 - \frac{3}{2}x^3 - \frac{27}{2}x^2 + x - 4$

Concave up: $(3,\infty)$

Concave down: $(-\infty,3)$

Point of inflection: $(3,-130.6)$

b) $y = 2x^{1/3} + 3$

Concave up: $(-\infty,0)$

Concave down: $(0,\infty)$

point of inflection: $(0,3)$

5. If f is continuous on [0, 3] and satisfies the following:

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3
$f(x)$	0	+	2	+	0	–	–2
$f'(x)$	3	+	0	–	DNE	–	–3
$f''(x)$	0	–	–1	–	DNE	–	0

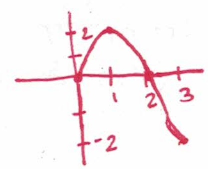
a) Find the absolute extrema of f and where they occur. Justify your response.

max: $(1,2)$ since f' changes signs at $(1,2)$

b) Find any points of inflection. Justify your response.

none, there are no points where y'' changes signs

c) Sketch a possible graph of f.



6. Let f be a function tha[illegible] ntinuous on the interval [0, 4]. The function f is twice differentiable except at $x = 2$. The function f [illegible] ts deri[illegible]ves have the properties indicated in the table above, where DNE indicates that the derivatives of f [illegible] not exist at $x = 2$.

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

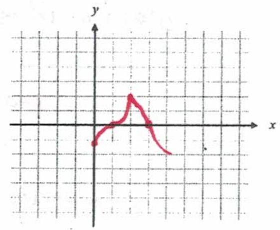
a) Describe the behavior of $f(x)$ in each interval using the information above.

x	$0<x<1$	$1<x<2$	$2<x<3$	$3<x<4$
$f(x)$	inc. conc. down	inc. conc. up	dec. conc. down	dec. conc. up

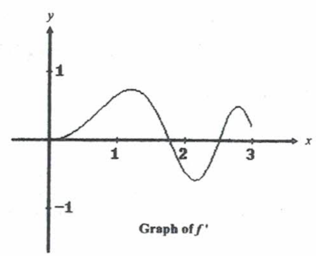
b) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

max: (2,2) since f' changes from positive to negative at (2,2)

c) On the axes provided, sketch the graph of a function that has all the characteristics of f.



7. [Calculator Required] Let f be a function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)}\sin(x^2)$. The graph of $y = f'(x)$ is shown below.



Graph of f'

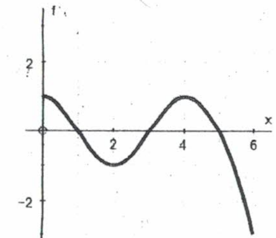
a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

concave down since f' is decreasing on 1.7< x <1.9

b) Write an equation for the line tangent to the graph of f at the point (2, 5.623).

y= -0.459x +6.541

Use the graph of $f'(x)$ defined on [0, 6] provided below to estimate the following:



a) When is f increasing? When is f decreasing? Justify your response.

inc. (0,1) ; (3,5) since f'>0

dec. (1,3) ; (5,6) since f'<0

b) Determine the x-coordinates of all local extrema. Justify your response.

x = 1 , x = 3 , x = 5 since f'(x) = 0

c) Determine when f is concave up and concave down. Justify your response.

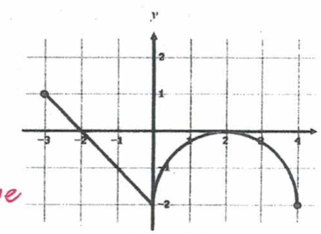
conc. up. (2,4) since f' is increasing

conc. down (0,2); (4,6) since f' is decreasing

d) Determine whether f has any points of inflection. Justify your response.

x=2 , x=4 since f" would change signs at these values

9. [No Calculator Allowed] Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown below.



Graph of f'

a) On what intervals, if any, is f increasing? Decreasing? Justify your answer.

inc: (-3,-2) since f'>0

dec: (-2,4) since f'<0

b) Find all values of x for which f assumes a relative maximum. Justify your answer.

x=-2 since f' changes from positive to negative

c) Where is the graph of f concave up? concave down? Justify your answers.

conc. up (0,2) since f' is increasing

conc. down (-3,-2); (2,4) since f' is decreasing

d) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

x=0 & x=2 f' changes from decreasing to increasing at x=0 and inc. to dec. at x=2

e) Find an equation for the line tangent to the graph of f at the point (0, 3).

y=-2x+3

f) Sketch a possible graph of f.